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THE THEORY OF THE 180° MAGNETIC FOCUSSING TYPE OF BETA RAY SPECTROMETER*

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ABSTRACT. A rigorous mathematical theory of the 180° magnetic focussing type of β-ray spectrometer has been worked out. An expression for the transmission factor of this instrument as a function of the magnetic field and the electron-momentum has been obtained. The transmission factor of the spectrometer of Lawson and Tyler has been completely calculated numerically in order to compare this theory with an earlier approximate theory developed by Lawson and Tyler.

§1. The 180° magnetic focussing type of β-ray spectrometer with variable field has become today, like the ionisation chamber, the Geiger Müller counter and the cloud chamber, an indispensable instrument in nuclear physics laboratories. It has found extensive application in the study of the shape of the continuous β-ray spectrum and the determination of the position and the intensity of the conversion electron groups. On account of the great importance of this apparatus it is necessary to have its complete mathematical theory. The only existing theory appears to be that due to Lawson and Tyler (1940). They have, in course of developing their theory, been forced to resort to many approximations and have employed graphical methods even at the initial stages of the theory. Consequently their mathematical working is rather obscure at many points. The following is an alternative treatment of this problem, being essentially a continuation of a method developed by the writer [Saha (1944)]† in connection with his theory of the screen cathode β-ray spectrometer. Almost the same notations, as used in the previous paper, have been employed here throughout. In this theory the method is strictly geometrical, and the analysis may be continued up to the final stages without employing any simplifying approximations. The permissible approximations come out automatically in course of the analysis and these have been clearly indicated in the final calculations of the so-called transmission factor function of this instrument. Again it must be pointed out, that only one graphical integration has to be carried out in this analysis and even that at the final stages, whereas Lawson and Tyler had to carry out more than one graphical integrations. Thus this method turns out, for practical purposes, to be simpler than that of Lawson and Tyler. Finally the method has been used to calculate the transmission factor function for the Lawson-Tyler β-ray spectrometer in order to compare our theory with that of Lawson and Tyler.

* Communicated by Prof. M. N. Saha, F.R.S., F.N.I.

† This Paper will be referred to here after as Paper I.

§2. MOTION OF THE ELECTRONS IN A MAGNETIC FIELD H

In fig. 1 O represents the centre of the rectangular plate over which the radioactive sample under investigation is spread. With this point as origin, let us take two axes of co-ordinates $O\xi$ and $O\eta$ as shown in the figure. $O\xi$ is perpendicular to the shorter edge of the sample plate and the positive direction of ξ points away from the reader. $O\eta$ is perpendicular to the longer edge of the sample plate and the positive direction of η points to the right side of the figure, *i.e.*, away from the slit S_1. Let S be a point (ξ, η) on the sample plate. At S we construct a right handed frame of reference as shown in the figure. SZ and SY are respectively parallel to $O\xi$ and $O\eta$. SX is normal to the plane of the sample plate and positive direction of x is upwards from the plate. The

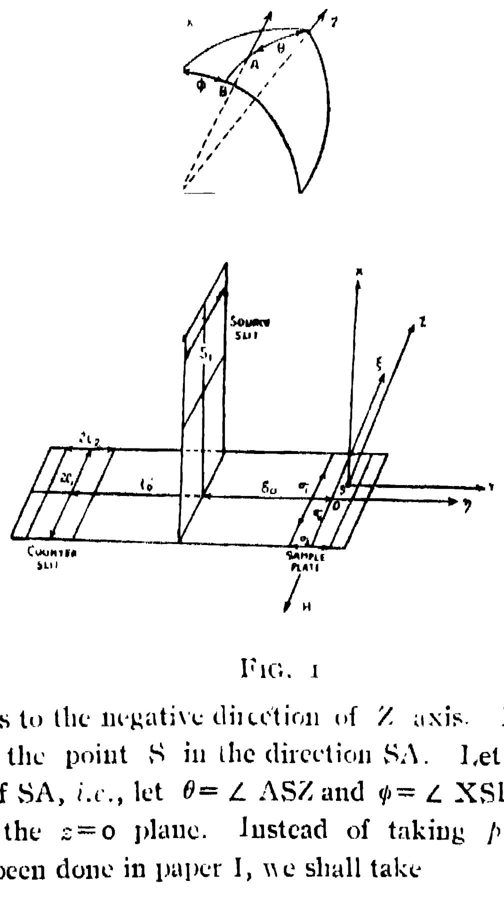


FIG. 1

magnetic field points to the negative direction of Z axis. Let us suppose that an electron leaves the point S in the direction SA. Let θ be the zenith angle and ϕ the azimuth of SA, *i.e.*, let $\theta = \angle$ ASZ and $\phi = \angle$ XSB, where SB is the projection of SA on the $z=0$ plane. Instead of taking p to be the electron momentum, as has been done in paper I, we shall take

$$p \quad \times \text{ electron momentum.} \qquad (2.1)$$

Let H be the magnetic field applied. As pointed out in paper I the electron trajectory has the shape of a helix whose axis is parallel to the magnetic field and whose projection on the $z=0$ plane is a circle. If the quantities D, ρ, L be defined by

$$D = \frac{2p}{H}, \quad \rho = \frac{D}{2}\sin\theta, \quad L = \pi D\cos\theta, \qquad \ldots \quad (2.1)$$

then the electron trajectory is given by

$$(x-\rho \sin \phi)^2+(y+\rho \cos \phi)^2=\rho^2, \quad \text{.. (2.2a)}$$

$$z=\frac{L\psi}{\pi} . \quad \text{... (2.2b)}$$

Equation (2.2a) represents the projection of the helix on the plane $z=0$. It is a circle of radius ρ, with its centre at N (fig. 2) whose co-ordinates are

$$x_N=\rho \sin \phi, \quad y_N=-\rho \cos \phi \quad \text{... (2.3)}$$

ψ is an angle which has been called in paper I by the name pitch angle. If E be the projection of any point on the electron trajectory on the $z=0$ plane then the corresponding ψ is the $\angle$ EFS. The significance of the length L, the pitch of the helix has been explained in paper I. For convenience we shall call:

(*a*) the circle (2.2a) as the circle P.

(*b*) the locus of centres of the circles of P for the same D and θ but different ϕ as the circle Q.

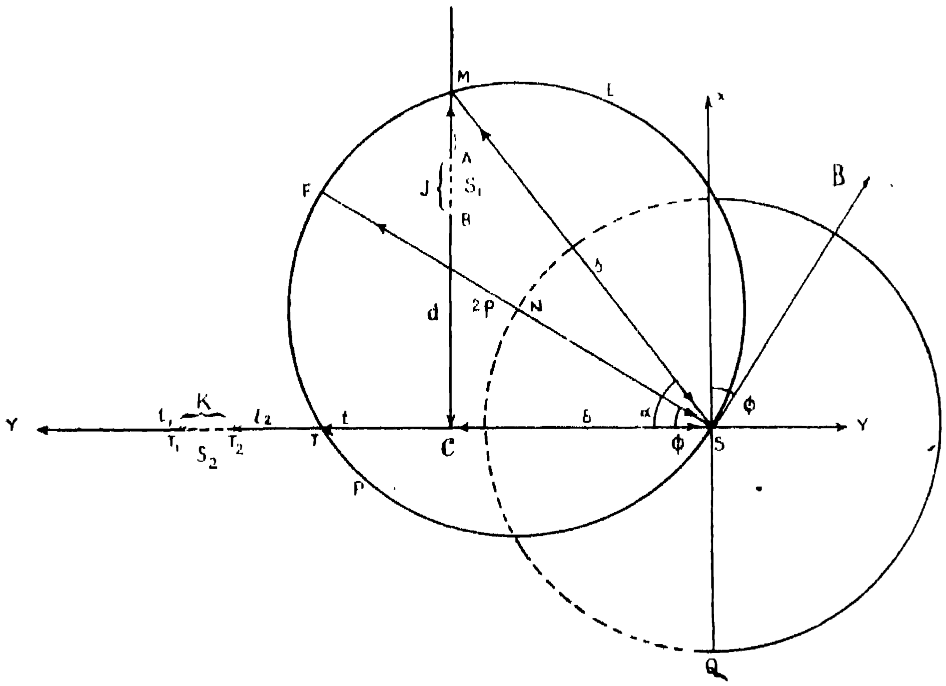


FIG. 2

Before proceeding any further, we shall explain certain symbols which we shall have to use very frequently hereafter. Consider the section of the spectrometer by the plane $z=0$. Let this plane cut the source slit S_1 at points A, B and the counter slit S_2 at points T_1, T_2. Let AB cut ST_1T_2 at C. Let the circle P cut the line AB at a point M and the line OC at a point T.

Let the symbols α, s, δ, t, d be defined as follows

$$\alpha = \angle \text{ MSC}, \quad s = \text{MS}, \quad \delta = \text{CS}, \quad t = \text{TS}, \quad d = \text{MC}.$$

Then
$$\frac{\sin\alpha}{d} = \frac{\cos\alpha}{\delta} = \frac{t}{s} \quad \ldots \quad (2.4)$$

If δ_0 denotes the perpendicular distance of the centre O of the sample plate from the plane of S_1 then

$$\delta = \delta_0 + \eta \quad \ldots \quad (2.5)$$

It is easy to prove that if Φ = the azimuth of an electron (p) for which the circle (2.2a) passes through a point M(α) on S_1, then

$$\Phi = \alpha + \sin^{-1}\left(\frac{s}{D\sin\theta}\right) - \frac{\pi}{2} \quad \ldots \quad (2.6)$$

The distance t for this electron is given by

$$t = 2p\cos\Phi = \delta + \sin\alpha\sqrt{D^2\sin^2\theta - s^2} \quad \ldots \quad (2.7)$$

The z-displacement of the electron when it reaches the counter slit is given by $z = Z$, where

$$Z = \frac{L}{\pi}\left(\frac{\pi}{2} + \Phi\right) = D\cos\theta\left[\alpha + \sin^{-1}\left(\frac{s}{D\sin\theta}\right)\right] \quad \ldots \quad (2.8)$$

The portion of the line AB extending from A to B will be called by the name J. By K we shall understand the portion of the line SY′ extending from T_1 to T_2. Let $z = \lambda_+, -\lambda_-$ and $y = -l_1, -l_2$ be the boundaries of the slit S_2. If $2C_1$ denotes the length of the longer edge of S_2 then

$$\lambda_+ = C_1 - \xi, \; \lambda_- = C_1 + \xi. \quad \ldots \quad (2.9a)$$

If $2C_2$ denotes the length of the shorter edge of S_2 and l_0 be the distance of the centre of the sample plate from the centre of counter slit then

$$l_1 = l_0 + \eta + C_2, \quad l_2 = l_0 + \eta - C_2 \quad \ldots \quad (2.9b)$$

The condition that an electron will be admitted by the counter slit S_2 and be counted are as follows :—

(a) The circle P should cut both the lines J and K ... (2.10a)

(b) (1) $0 < Z < \lambda_+$ for electrons with θ in $0 < \theta < (\pi/2)$ ... (2.10b)

(2) $\lambda_- > Z < 0$ for electrons with θ in $+(\pi/2) < \theta < \pi$ ... (2.10c)

As pointed out in paper I electrons with θ in $2\pi > \theta > \pi$ or ϕ in $3\pi/2 > \phi > \pi/2$ will be cut off by the source plate. We have therefore the following restrictions on ϕ and θ, *viz.*

$$\left.\begin{array}{ll}(a) & \phi \text{ can lie only in the interval } \frac{\pi}{2} > \phi > -\frac{\pi}{2} \\ (b) & \text{and } \theta \text{ within } \pi > \theta > 0\end{array}\right\} \quad \ldots \quad (2.11)$$

§3. RESTRICTIONS ON ϕ IN ORDER THAT AN ELECTRON (p) MAY PASS THROUGH S_1

The angle α can vary only between the limits α_A, α_B (these being the α's corresponding to the points A and B respectively), for $\alpha_A > \alpha > \alpha_B$ is the only open portion on S_1. Thus Φ can vary only between the limits $\Phi_A > \Phi > \Phi_B$ (Φ_A, Φ_B being the Φ's corresponding to electrons p whose M coincide with the points A and B respectively). Let us now examine how Φ for any particular α varies with 2ρ. The Φ v.s. 2ρ plot for any particular α [fig. 4(*a*)] starts at ($2\rho = s_\alpha$, $\Phi = \alpha$) and falls uniformly as we increase 2ρ. Again $\Phi(\alpha) = 0$ at

$$2\rho = s_\alpha \sec \alpha \tag{3.1}$$

Thus for $s < 2\rho < s \sec \alpha$, we have $\Phi_\alpha > 0$ and for $2\rho > s \sec \alpha$ we have $\Phi_\alpha < 0$. Further more as $2\rho \to \infty$, $\Phi(\alpha) \to \alpha - \pi/2$ asymptotically. Suppose that at $\rho = \rho_{AB}$ $\Phi_A = -\Phi_B$. ρ_{AB} will be given by

$$2\rho_{AB} = \frac{s_A s_B}{\delta} \tag{3.2}$$

Let us now examine the following cases in some detail.

Case I: $s_A < s_B \sec \alpha_B$ (figures 3).

(1) $s_B < 2\rho < s_A$:—The circles P and the line J will cut at points between D [$\alpha = \alpha_D = \cos^{-1}(\delta/2\rho)$] and B ($\alpha = \alpha_B$) provided $\Phi_D (= \alpha_D) > \phi > \Phi_B$. *i.e.* provided N is on the arc $N_D N_B$ [fig. 3(*a*)] of the circle Q. Both Φ_D and Φ_B are

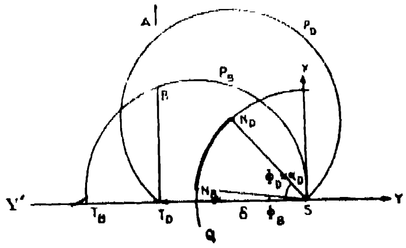


FIG. 3(*a*)

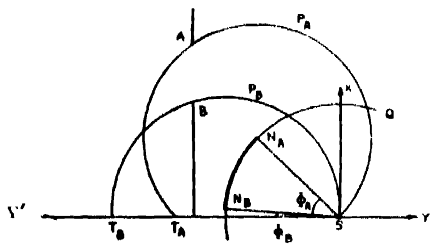


FIG. 3(*b*)

> 0 and therefore the arc $N_D N_B$ is above SY' so that according to (2.7) $t_B > t > t_D$.

(2) $s_A < 2\rho < s_B \sec \alpha_B$:—The circles P and the line J will cut provided $\Phi_A > \phi > \Phi_B$, *i.e.*, provided N is on the arc $N_A N_B$ [fig. 3(*b*)] of the circle Q. Φ_A and Φ_B are both > 0, so that the arc $N_A N_B$ [fig. 3(*b*)] is above SY'. Hence $t_B > t > t_A$.

(3) $s_B \sec \alpha_B < 2\rho < 2\rho_{AB}$:—$\Phi_A > 0$, $\Phi_B < 0$ and $\Phi_A > -\Phi_B$. The circles P and the line J will cut if $\Phi_A > \phi > \Phi_B$, *i.e.*, provided N is on the arc $N_A N_B$. The position of the centres N_A, N_B of the circles P_A and P_B relative to the line SY' is as shown in fig. 3(*c*). Let N_C be the centre of the circle P_C for an electron p whose $\Phi = 0$. The α of the point where P_0 cuts J is given by

$$\alpha_C = \cos^{-1}(\delta/2\rho) \qquad \dots \tag{3.3}$$

Evidently then $t_C > t > t_A$. We must note an important fact here. If the centres N_{E_1}, N_{E_2} of two circles P_{E_1} and P_{E_2} are so placed on the arc N_AN_B, one above, and the other below SY′, that arc $N_CN_{E_1}$ = arc $N_CN_{E_2}$, then since $\Phi_{E_1} = -\Phi_{E_2}$, we find that the circles P_{E_1} and P_{E_2} will intersect each other at a point T_E which lies on the line SY′. Thus $t_{E_1} = t_{E_2}$ [fig. 3(c)].

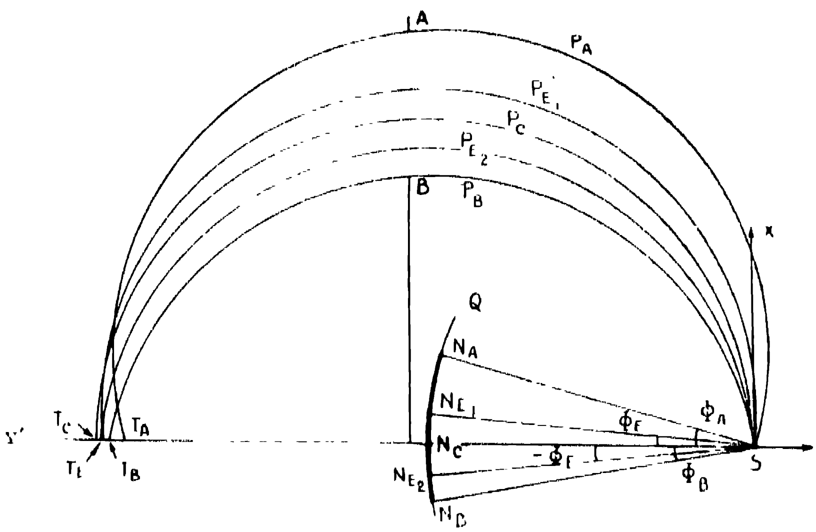


FIG. 3(c)

(4) $2\rho_B < 2\rho < s_A \sec \alpha_A$:—$\Phi_A > 0$, $\Phi_B < 0$, and $\Phi_A < -\Phi_B$. In this case also N must lie on the arc N_AN_B if the circles P are to cut the line J [fig. 3(d)]. But since now $\Phi_A < -\Phi_B$ we shall have $t_C > t > t_B$ [fig. 3(d)]. The rest of the discussion in (3) hold here also.

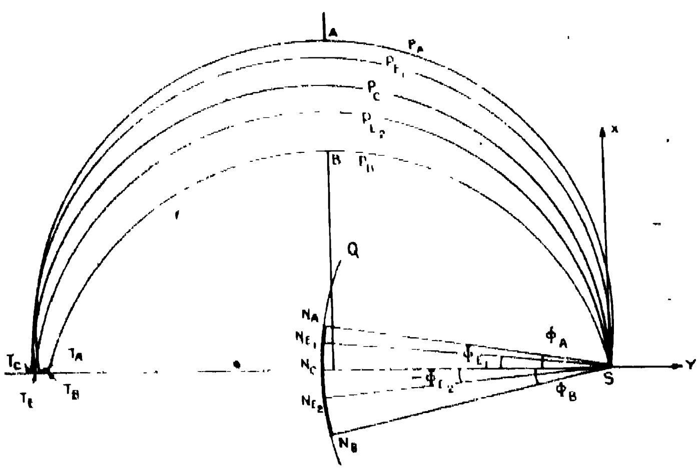


FIG. 3(d)

(5) $s_A \sec \alpha_A < 2\rho < \infty$:—Φ_A and Φ_B are both < 0, and $-\Phi_B > -\Phi_A$. Again

N must lie on the arc N_AN_B if the circles P are to cut the line J. The arc N_AN_B lie wholly below SY′ as shown in fig. 3e so that $t_A > t > t_B$.

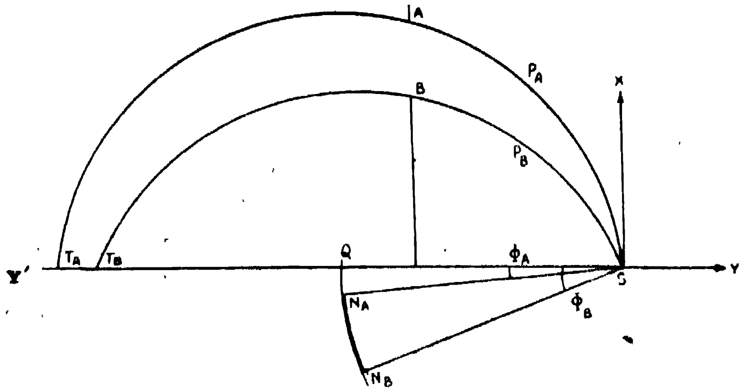


FIG. 3(e)

Case II : $s_A > s_B \sec \alpha_B$. This case needs no separate discussion. The conclusions are only trivially different from those of case I.

§4 FURTHER RESTRICTIONS ON ϕ INTRODUCED BY THE COUNTER SLIT S_2

Let us plot t against 2ρ for different α's and consider in detail the curves thus obtained. These are shown in fig. 4(b). From (2.7) it is clear that for any definite value of α : (a) t does not exist for $2\rho < t$, (b) at $2\rho = s_a$, $t = \delta$, (c) $dt/d(2\rho) = 1$ at $2\rho = s \sec \alpha$, (d) t increases uniformly as 2ρ is increased and (e) as $2\rho \to \infty$, $t \to$ asymptotically to the value $t = \delta + 2\rho \sin \alpha$, which represents the straight line RM_3 [fig. 4(b)] passing through ($2\rho = 0$ and $t = \delta$) and inclined to the 2ρ axis at an angle $\tan^{-1}(\sin \alpha)$. Thus for any particular point M on the line J with the source angle $\alpha(\alpha' > \alpha > \alpha_B)$, the t v.s. 2ρ curve starts at $M_1(2\rho = s_a$, $t = \delta)$, rises uniformly, touches a line B_2A_2 passing through ($t = 0$, $2\rho = 0$) and inclined to 2ρ axis an angle of $\pi/4$ at a point M_2 ($2\rho = s_a \sec \alpha = t$) and finally approaches asymptotically the line RM_3. The curves corresponding to points A, B on the line J are the curves $A_1A_2A_3$, $B_1B_2B_3$ respectively of the figures. Evidently these curves intersect at N ($2\rho = 2\rho_{AB}$, $t = t_{AB}$). Consider the curves $B_1\ B_2\ A_2\ A_3$ and $B_1\ A_1\ N\ B_3$. Clearly for any particular value of 2ρ the ordinate of the corresponding point on the former curve gives the maximum possible value of t for the circles P interesting the line J, and that of the corresponding point on the latter curve gives the minimum possible value of t. We may call therefore these curves by the name the t_{max} and the t_{min} curves respectively. In is easy to see that the t_{max} curves coincides (a) with , or $s_B < 2\rho < s_B \sec \alpha_B$, (b) with the line B_2A_2 (whose equation is $t = 2\rho$) for s_B s $\alpha_B < 2\rho < s_A \sec \alpha_A$ and (c) with t_A for $s_A \sec \alpha_A < 2\rho$. Also the t_{min} curve coincides with (a) the line B_1A_1 which is parallel to the 2ρ axis and is placed at a distance δ from it for $s_B < 2\rho < s_A$, (b) with t_A for $s_A < 2\rho < 2\rho_{AB}$, and

(c) with t_B for $2\rho_{AB} < 2\rho$. In passing, we note that in the regions $s_B < 2\rho < s_B \sec \alpha_B$ and $s_A \sec \alpha_A < 2\rho$, the t_α curves do not cut each other. But in the region $s_B \sec \alpha_B < 2\rho < s_A \sec \alpha_A$ each t_α curve cuts any other t_α curve once.

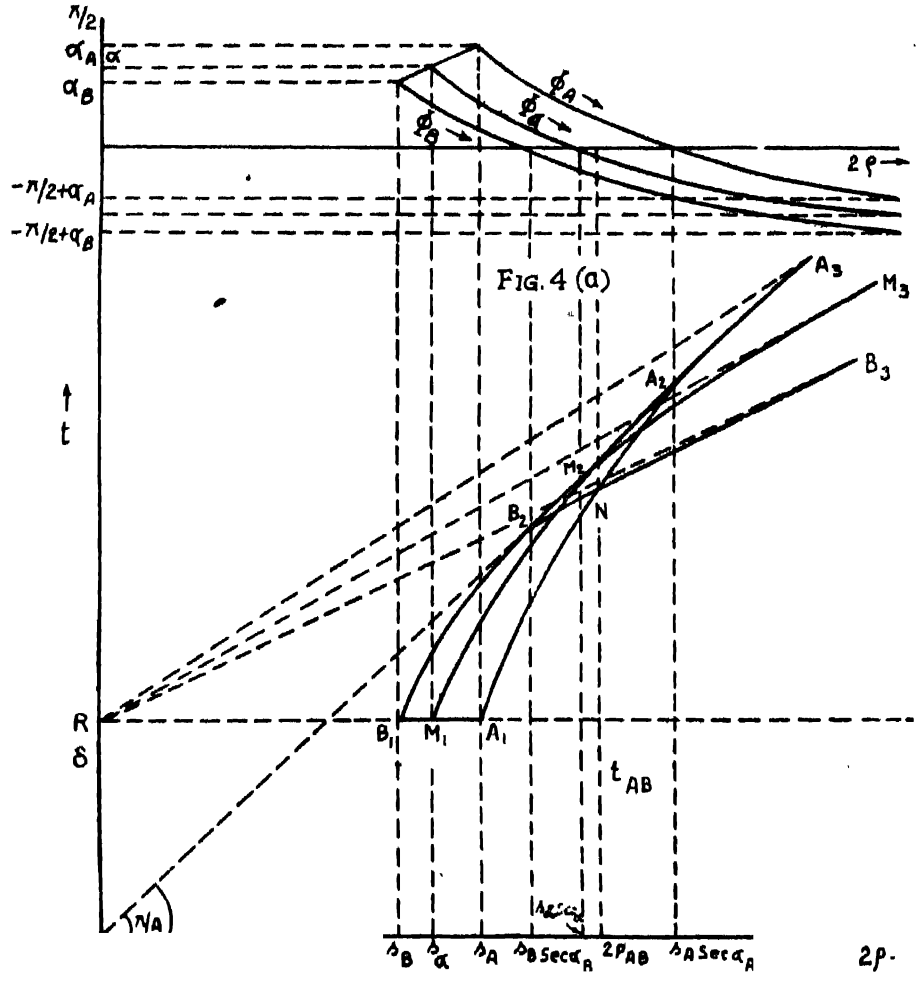


FIG. 4 (b)

Now let us draw two lines L_1 and L_2 parallel to and at distances l_1, l_2 respectively from the 2ρ axis. Let these lines cut the t_{max} curve at the points P_1 and P_2 respectively and the t_{min} curve at the points Q_1 and Q_2 respectively (figs. 5). Let the abscissae corresponding to these points be $2R_l$, $2R_s'$, $2R_l'$, $2R_s$ respectively. Then conditions (2.10a) show that the first condition that must be satisfied by ρ in order that the circles P may cut both the lines J and K is

$$2R_l < 2\rho < 2R_s \qquad \ldots \quad (4.1)$$

For each point S on the source plate these values R_l and R_s are constants and it is our purpose to determine them. Suppose that a line L parallel to and at a distance l from the 2ρ axis cuts the t_{max} curve at $2\rho = 2R_{II}$ and the t_{min} curve at $2R_{III}$. It is clear that (*a*) if $l < \delta$ then $R_{l\text{II}}$ does not exist,

(*b*) if $\delta < l < s_B \sec \alpha_B$, then $R_{l\text{II}}$ is given by

$$4R_{l\text{II}}^2 = \left(\frac{l-\delta}{\sin \alpha_B}\right)^2 + s_B^2,$$

(*c*) if $s_B \sec \alpha_B < l < s_A \sec \alpha_A$ then $R_{l\text{II}}$ is given by ... (4.2a)

$$2R_{l\text{II}} = l,$$

(*d*) if $s_A \sec \alpha_A < l$ then $R_{l\text{II}}$ is given by

$$4R^2_{l\text{II}} = \left(\frac{l-\delta}{\sin \alpha_A}\right)^2 + s_A^2.$$

Again we note that

(*a*) if $l < \delta$ then $R_{t\text{II}}$ does not exist,

(*b*) if $\delta = l$ then $R_{t\text{II}}$ may have all values between s_B and s_A,

(*c*) if $\delta < l < t_{AB}$ then $R_{t\text{II}}$ is given by

$$4R_{t\text{II}}^2 = \left(\frac{l-\delta}{\sin \alpha_A}\right)^2 + s_A^2,$$

(*d*) if $t_{AB} < l$ then $R_{t\text{II}}$ is determined by ... (4.2b)

$$4R_{t\text{II}}^2 = \left(\frac{l-\delta}{\sin \alpha_B}\right)^2 + s_B^2.$$

The quantity t_{AB} occurring above is the ordinate of the point N [fig. 4(*b*)] where the t_B and the t_A curves intersect. It is evidently given by

$$t_{AB} = \delta + \frac{d_A d_B}{\delta} \qquad \dots \quad (4.3)$$

Thus the quantities

$$R_t = R_{l_2\text{I}}, \quad R_s' = R_{l_2\text{II}}, \quad R_t' = R_{l_1\text{I}}, \quad R_s = R_{l_1\text{II}}, \qquad \dots \quad (4.4)$$

may be determined by the use of the appropriate formula given above.

We are now in a position after all these foregoing discussions to examine in detail the restrictions that should be imposed on φ in order that a circle P may intersect the lines J and K, the radius of the circle P being such that the condition (4.1) is satisfied. We have two distinct cases to consider.

Case I : It may happen that $R_t' > R_s'$ [fig. 5(*a*)].

(A) Let $2R_t < 2\rho < 2R_s'$. Let the abscissa of the point U_1 be 2ρ. Through U_1 draw the ordinate and let it cut t_{max} at V_1 and L_2 at W_1. Then

(A—1) :—if $t_{max}=t_B$, there will be only one t_a curve through W_1 and the azimuth of the circle P passing through this point α on the line J and having $t=l_2$ is

$$\Phi l_2=\cos^{-1}(l_2/2\rho). \quad \ldots \quad (4.5)$$

The circles P will cut both J and K if

$$\Phi l_2>\phi>\Phi_B. \quad \ldots \quad (4.6)$$

(A—2) :—If $t_{max}=2\rho$ then there will be two t_a curves through W_1 and the azimuths of the circles P passing through these points α on the line J are $\pm\Phi l_2$. The circles P will cut both J and K if

$$\Phi l_2>\phi>-\Phi l_2, \quad \ldots \quad (4.7)$$

provided both the t_A and t_B curves are below the line L_2. If however, one of these curves say t_A fall above this line, then $\Phi_A<\Phi l_2$ and consequently ϕ should lie between

$$\Phi_A>\phi>\Phi l_2. \quad \ldots \quad (4.8)$$

Similarly if t_B falls above L_2, ϕ should lie between

$$\Phi l_2>\phi>\Phi_B. \quad \ldots \quad (4.9)$$

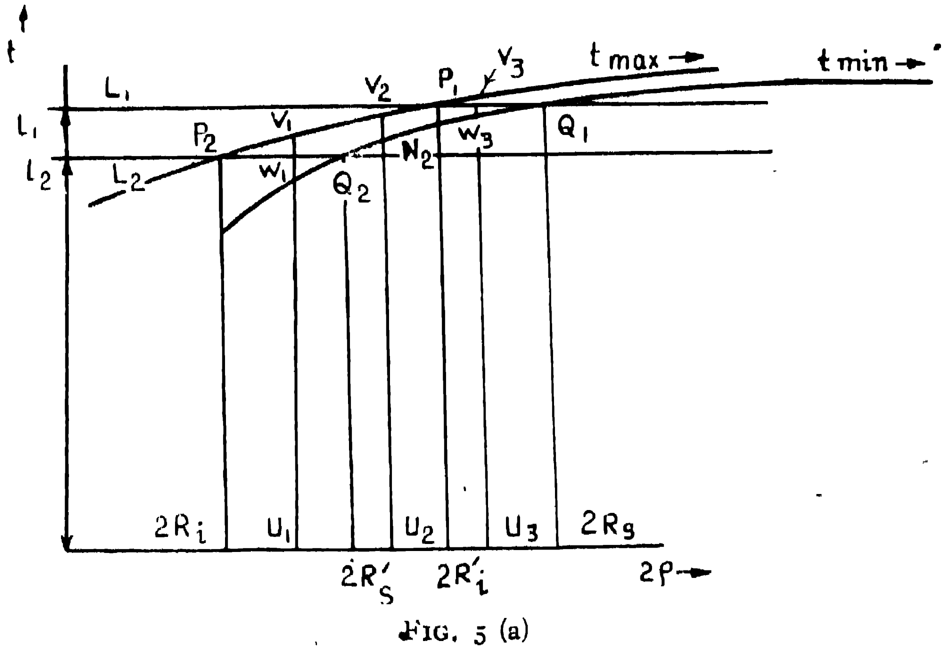


FIG. 5 (a)

(A—3) :—If $t_{max}=t_A$, then again there will be one t_a curve through W_1 and the corsesponding azimuth is $-\Phi l_2$. The circles P cut both the lines J and K if

$$\Phi_A>\phi>-\Phi l_2. \quad (4.10)$$

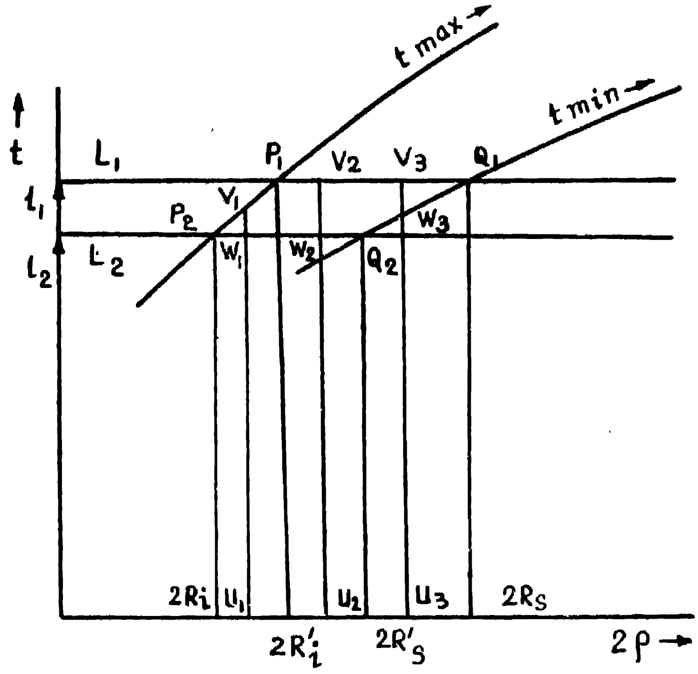


FIG. 5(*b*)

(B) Let $2R_s' < 2\rho < 2R_i'$. Let the abscissa of the point U_2 [fig. 5(*a*)] be 2ρ. Through U_2 draw the ordinate to cut t_{max} at V_2 and t_{min} at W_2. Then it can be easily seen by an examination of figs. 5*a*, 5*b* that no matter what t_{max} or t_{min} curves are, the circles P will cut both J and K if

$$\Phi_A > \phi > \Phi_B. \tag{4.11}$$

(C) Let $2R_i' < 2\rho < 2R_s$. Let the abscissa of the point U_3 [fig. 5(*a*)] be 2ρ. Through U_3 let us draw the ordinate to cut the line L_1 at V_3 and the t_{min} curve at W_3. Then if t_{max} curve is either the curve t_A or the curve t_B we find that there is only one t_a curve through V_3 and the azimuth of the circle P passing through this point (α) on the line J and having $t = l_1$ is $\pm \Phi l_1$ according as $t_{max} = t_B$ or t_A, the angle Φl_1 being defined by

$$\Phi l_1 = \cos^{-1} (l_1 / 2\rho). \tag{4.12}$$

On the other hand if $t_{max} = 2\rho$, then there are two t_a curves through W_3 and the azimuths of the two circles P passing through these two points on the line J but meeting at the same point $t = l_1$ on the line K are $\pm \Phi l_1$. Thus the circles P will cut both J and K in the case

$$(C-1)\, t_{max} = t_B \quad \text{and} \quad t_{min} = t_A, \quad \text{if } \Phi l_1 > \phi > \Phi_B, \qquad \ldots \tag{4.13}$$

$(C-2)\, t_{max} = 2\rho$ and $t_{min} = t_A$, if ϕ lies on either of the intervals

$$\Phi_A > \phi > \Phi l_1 \quad \text{or} \quad -\Phi l_1 > \phi > \Phi_B, \qquad \ldots \tag{4.14}$$

provided t_B lies below L_1. If however t_B lies above L_1, then the proper limits are

$$\Phi_A > \phi > \Phi l_1 \qquad \dots \quad (4.15)$$

(C − 3) $t_{max} = 2p$ and $t_{min} = t_B$, if ϕ lies on either of intervals

$$\Phi_A > \phi > \Phi l_1, \quad \text{or} \quad -\Phi l_1 > \phi > \Phi_B, \qquad \dots \quad (4.16)$$

provided t_A lies below L_1. If however t_A lies above L_1, then the proper limits are

$$-\Phi l_1 > \phi > \Phi_B \qquad \dots \quad (4.17)$$

(C − 4) $t_{max} = t_A$ and $t_{min} = t_B$, if $-\Phi l_1 > \phi > \Phi_B$. ... (4.18)

Case II : Again it may happen that $R_i' < R_s$ [fig. 5(*b*)].

(A) If $2R_i < 2p < 2R_i'$ then the discussion is the same as that of (I.A).

(B) Let $2R_i' < 2p < 2R_s'$ Let the abscissa of the point U_2 be $2p$. Draw the ordinate through U_2 to cut the lines L_1 and L_2 at V_2 and W_2 respectively. Then as in case I, if t_{max} is either t_B or t_A then only one t_a curve passes through V_2 and another through W_2 and the azimuths of the circles P passing through the corresponding points on the line J and meeting the line K at $t = l_1$ and $t = l_2$ are

$$\Phi l_1,\ \Phi l_2 \text{ respectively if } t_{max} = t_B,$$

or

$$-\Phi l_1 - \Phi l_2 \text{ respectively if } t_{max} = t_A.$$

On the other hand if $t_{max} = 2p$, then two t_a curves pass through V_2 and another pair through W_2. Thus azimuths of the two circles belonging to the family P which meet the line K at $t = l_1$ are $\pm\Phi l_1$ and those of the two circles P meeting the line K at $t = l_2$ are $\pm\Phi l_2$. Thus circle P will cut both J and K in the case when

(B − 1) $t_{max} = t_B$, if ϕ lies on $\Phi l_2 > \phi > \Phi l_1$. ... (4.19)

(B − 2) $t_{max} = 2p$, $t_{min} = t_A$, if ϕ lies on either

$$\Phi l_2 > \phi > \Phi l_1 \quad \text{or} \quad -\Phi l_1 > \phi > -\Phi l_2 \qquad (4.20)$$

provided t_B lies below L_2. If t_B lies above L_1 then the interval on which ϕ should lie is

$$\Phi l_2 > \phi > \Phi l_1. \qquad \dots \quad (4.21)$$

Again if t_B lies between L_1 and L_2, then the intervals are

$$\Phi l_2 > \phi > \Phi l_1 \quad \text{or} \quad -\Phi l_1 > \phi > \Phi_B. \qquad \dots \quad (4.22)$$

(B − 3) $t_{max} = 2p$, $t_{min} = t_B$, if ϕ lies on either

$$\Phi l_2 > \phi > \Phi l_1 \quad \text{or} \quad -\Phi l_1 > \phi > -\Phi l_2 \qquad \dots \quad (4.23)$$

provided t_A lies below L_2. If t_A lies above L_2 then the proper interval is

$$-\Phi l_1 > \phi > -\Phi l_2. \qquad \dots \quad (4.24)$$

Again if t_A lies between L_1 and L_2, then the intervals are

$$\Phi_A > \phi > \Phi_{l_1} \quad \text{or} \quad -\Phi_{l_1} > \phi > -\Phi_{l_2}. \tag{4.25}$$

(B–4) $t_{max} = t_A$, if ϕ lies on $-\Phi_{l_1} > \phi > \Phi_{l_2}$. (4.26)

(C) If $2R_s' < 2\rho < 2R_s$, then the discussion is the same as that of (I-C).

This discussion, given in this section, is too abstract and tedious, and one may lose the general trend of the method in the maze of mathematical formulae appearing above. So we propose, before proceeding any further, to take up the concrete case of the Lawson-Tyler β-ray spectrometer, and illustrate the use of our method. For this instrument we have

$$2\delta_0 = 24 \text{ cm}, \qquad \sigma_1 = 2C_1 = 1.6 \text{ cm}, \qquad \sigma_2 = 2C_2 = .3 \text{ cm},$$

$$d_A = 13.89 \text{ cm}, \qquad d_B = 10.11 \text{ cm}.$$

Let us take $\eta = +.15$ cm. Then the values of the corresponding quantities δ, l_2, l_1, etc. appear in the following Table.

TABLE I

η	δ	$l_2 = 2R_i$	$l_1 = 2R_i'$	α_B	α_A	s_B	s_A	$s_B \sec \alpha_B$	$s_A \sec \alpha_A$	l_{AB}	$2R_s'$	$2R_s$	$2R_{Al_2}$	$2R_{Al_1}$
.15 cm	12.15 cm	24 cm	24.3 cm.	.6940 rad.	.8525 rad	15.81 cm	18.48 cm	20.68 cm	28.05 cm.	23.71 cm	24.35 cm.	24.71 cm	24.27 cm	24.53 cm.

Since $s_B \sec \alpha_B < l_2 < l_1 < s_A \sec \alpha_A$, it is clear that for the region we are interested in, $t_{max} = 2\rho$. Consequently $2R_i$, $2R_i'$ are given by formula (4.2a) [case (c)], being therefore respectively l_2 and l_1. Again since $l_{AB} < l_2 < l_1$, $2R_s'$, $2R_s$ are respectively given by formula (4.2b) [case (d)]. Evidently, $2R_i' < 2R_s'$ and therefore we have to discuss a figure like fig. 5(b). Let t_A curve cut the lines L_1, L_2 respectively at the points $2\rho = 2R_{Al_1}$ and $2R_{Al_2}$ respectively. These are determined by

$$(2R_{Al_1})^2 = (l_1 - \delta/\sin \alpha_A)^2 + s_A^2, \quad (2R_{Al_2})^2 = (l_2 - \delta/\sin \alpha_A)^2 + s_A^2.$$

The actual numerical values of these quantities are given in Table I. We shall have later occasion to calculate $T_1(\rho, \eta)$, the total length of domain over which the azimuth ϕ should lie in order that the circles P may cut both the lines J and K. In what follows we shall obtain expressions for $T_1(\rho, .15)$. Thus for

(*a*) $2R_i < 2\rho < 2R_{Al_2}$, the azimuth should lie between $\Phi_{l_2} < \phi < -\Phi_{l_2}$ [*cf.* II.A above, *i.e.*, formula (4.7)]. Thus $T_1(\rho, .15) = 2\Phi_{l_2}$.

(*b*) $2R_{Al_2} < 2\rho < 2R_i'$, the range on which the azimuth should lie is $\Phi_A > \phi > -\Phi_{l_2}$ [*cf.* formula (4.8)]. Thus $T_1(\rho, .15) = \Phi_A + \Phi_{l_2}$.

(*c*) $2R_i' < 2\rho < 2R_s'$, the intervals are $\Phi_A > \phi > \Phi_{l_1}$ and $-\Phi_{l_1} > \phi > -\Phi_{l_2}$ [*cf.* formula (4.26)]. Thus $T_1(\rho, .15) = \Phi_A + \Phi_{l_2} - 2\Phi_{l_1}$.

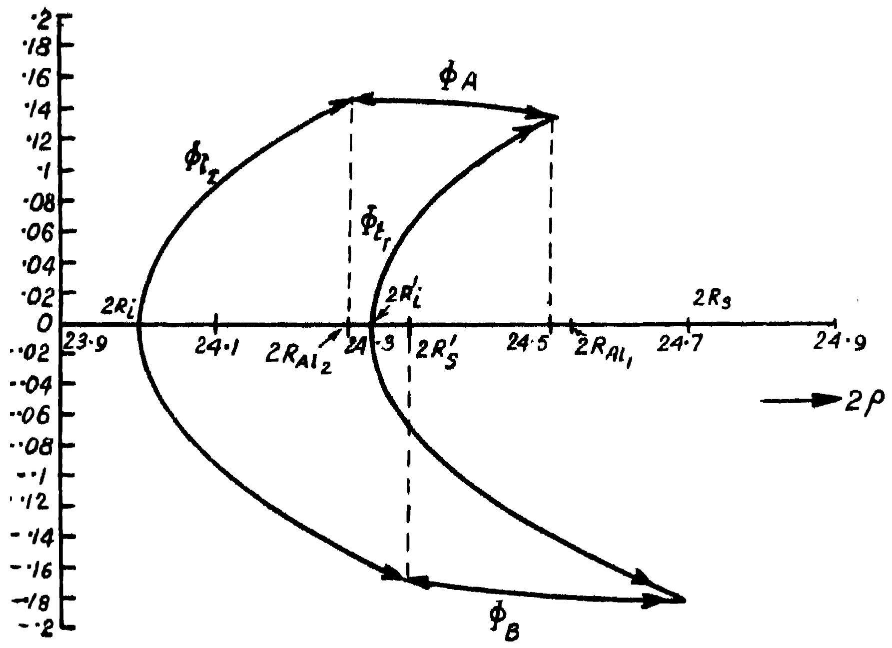


Fig. 6(a)

(d) $2R_s' < 2\rho < 2R_{Al_1}$, the intervals are $\Phi_A > \varphi > \Phi_{l_1}$, $-\Phi_{l_1} > \varphi > \Phi_B$ [*cf.* formula (4.17)]. Thus $T_1(\rho, .15) = \Phi_A - \Phi_B + 2\Phi_{l_1}$.

(e) $2R_{Al_1} < 2\rho < 2R_s$, the range is $-\Phi_{l_1} > \varphi > \Phi_B$ [*cf.* formula (4.18)]. Thus $T_1(\rho, .15) = -\Phi_{l_1} - \Phi_B$.

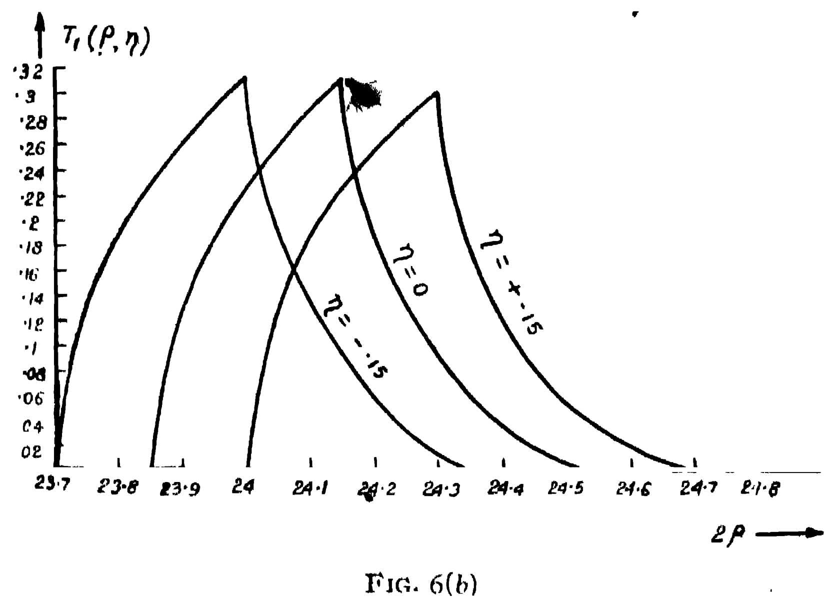


Fig. 6(b)

This discussion should be followed with the help of fig. 6(*a*). The function $T_1(\rho, .15)$ has been plotted against 2ρ in fig. 6(*b*). We may carry out the whole procedure given above, for $\eta = 0$ cm, and $-.15$ cm with necessary modifications. The corresponding curves $T_1(\rho, \eta)$ are shown in fig. 6(*b*). It is clear that forms of these curves do not depend much on η. In fact, the $T_1(\rho, \eta)$ curves for different values of η may be obtained by shifting $T_1(\rho, 0)$ curve by an amount η. Thus we may take in future

$$T_1(D, \eta) = T_1(D - \eta, \eta = 0). \quad \ldots \quad (4.27)$$

This completes the discussion of the restrictions necessary on the azimuth of an electron in order that it may pass through the counter slit. These restrictions are introduced by the finite extensions perpendicular to the magnetic axis of the slits S_1 and S_2. The extensions of these slits parallel to the z-axis, *i.e.*, to the magnatic field introduces certain restrictions on θ and D so that conditions (2.10) are satisfied. These restrictions will be calculated in what follows.

§5. RESTRICTIONS THAT MUST BE IMPOSED ON θ

We note that the z-displacement of the point where an electron cuts the counter slit S_2 is never less than the z-displacement of the point where this electron cuts the source slit. So it is enough to calculate what restrictions are necessary on the parameters θ and D in order that an electron may pass through the counter slit. We shall confine our attention first to the positive portion of S_2 where $0 < \theta < \pi/2$.

As pointed out already in §2, the z-displacement of an electron when it reaches the slit S_2 is Z, where Z is given by (2.8). Clearly Z will exist only if 2ρ, *i.e.*, $D \sin \theta < s$. Thus if $D > s$ and θ_1 is defined by

$$\sin \theta_1 = s/D, \quad \ldots \quad (5.1)$$

then Z exists if $\theta < \theta_1$. Evidently

$$Z[\theta_1(\alpha)] = (\alpha + \frac{\pi}{2}) \sqrt{D^2 - s^2}\ . \quad \ldots \quad (5.2)$$

Now $\theta_1(\alpha_A) > \theta_1(\alpha) > \theta_1(\alpha_B)$ for $\alpha_A > \alpha > \alpha_B$. For $\theta > \theta_1(\alpha_A)$ it is clear that $Z(\theta, \alpha_A) > Z(\theta, \alpha) > Z(\theta, \alpha_B)$. The discussion of the $Z(\theta, \alpha)$ curves for $\theta < \theta_1(\alpha_A)$ is difficult but is fortunately unnecessary as we shall see presently we shall have almost always to discuss θ's which are $> \theta_1(\alpha_A)$.

It is clear that a $Z(\theta)$ curve for any particular α and D starts from a maximum at $\theta = \theta_1(\alpha)$, then falls uniformly as we increase θ and reaches the zero value at $\theta = \pi/2$. [*cf.* fig. 7(*a*)].

We have seen already that in order that conditions (2.10) may be satisfied we must have condition (4.1) satisfied. If the angles θ_i, θ_i', θ_s', θ_s be defined by

$$\sin \theta_i = \frac{2R_i}{D}, \quad \sin \theta_i' = \frac{2R_i'}{D}, \quad \sin \theta_s' = \frac{2R_s'}{D}, \quad \sin \theta_s = \frac{2R_s}{D}, \ldots \quad (5.3)$$

then

$$
\begin{aligned}
&(a)\ \text{if } D \gtrsim 2R_i, \quad \text{we have } 2\rho \gtrless 2R_i \ \text{ according as } \theta \gtrless \theta_i\,, \\
&(b)\ \text{if } D > 2R_i', \quad \text{we have } 2\rho \gtrless 2R_i' \ \text{ according as } \theta \gtrless \theta_i', \\
&(c)\ \text{if } D > 2R_s' \quad \text{we have } 2\rho \gtrless 2R_s' \ \text{ according as } \theta \gtrless \theta_s', \\
&(d)\ \text{if } D \gtrsim 2R_s \quad \text{we have } 2\rho \lessgtr 2R_s \ \text{ according as } \theta \gtrless \theta_s\,.
\end{aligned}
\tag{5.4}
$$

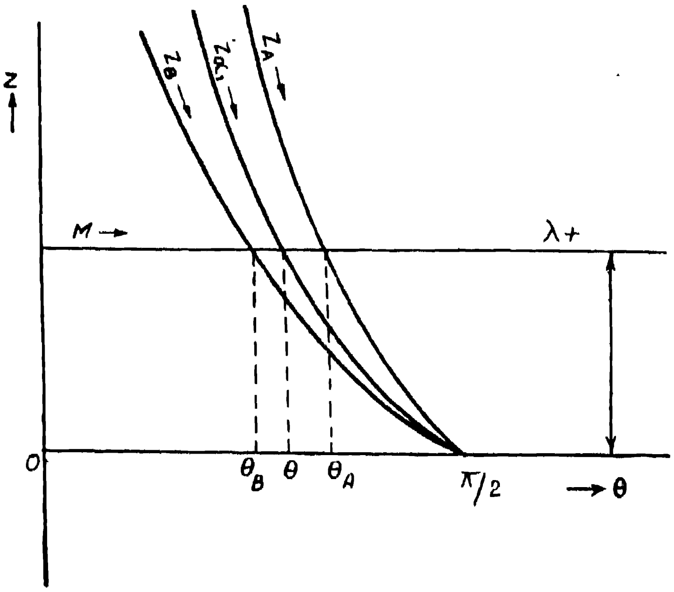


FIG. 7(*a*)

In fig. 7(*b*) we have plotted the angles $\theta_1(\alpha)$, θ_i, θ_s, as functions of D. $\theta_1(\alpha_B)$, $\theta_1(\alpha_A)$ v.s. D curves start respectively at $[D=s_B,\ \theta_1(\alpha_B)=\pi/2]$, $[D=s_A,$

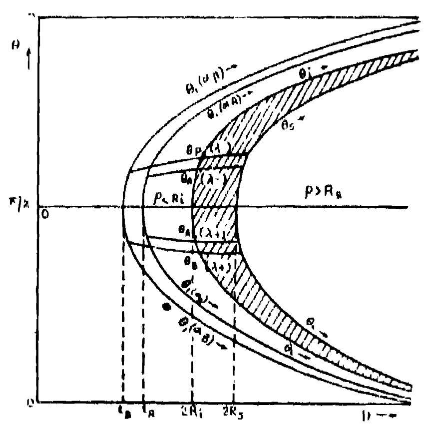


FIG. 7(*b*)

$\theta_1(\alpha_A)=\pi/2$] and falls uniformly as D is increased. The θ_i, θ_i', θ_s', θ_s curves also start from the maximum value $\pi/2$ at $D=2R_i$, $2R_i$, $2R_s'$, $2R_s$ respectively and fall uniformly as D is increased. Again since in almost all cases $2R_i > s_A$ we must have $\theta_s > \theta_i > \theta_1(\alpha_A) > \theta_1(\alpha_B)$ for $D > 2R_i$. From the figure it is clear that the condition (4.1) is satisfied only when the point (D, θ) lies on the shaded area bounded by $\theta=\pi/2$ line, $\theta=\theta_i$ and $\theta=\theta_s$ curves.

We shall now calculate for what θ will the curves $Z(\theta)$ for any particular D and α, cut the line M which is parallel to and at a distance λ_+ from the θ-axis [fig. 7(b)]. Let us suppose that the two $Z(\theta)$ curves correspondig to $\alpha=\alpha_A$ and $\alpha=\alpha_B$ cut the line M respectively at $\theta=\theta_A$ and $\theta=\theta_B$. To determine these angles we must solve the transcendental equations

$$\left.\begin{aligned} D \sin\theta \sin\left[\frac{\lambda_+ \sec\theta_A}{D} - \alpha_A\right] &= s_A \\ D \sin\theta_B \sin\left[\frac{\lambda_+ \sec\theta_B}{D} - \alpha_B\right] &= s_B \end{aligned}\right\} \quad \cdots \quad (5.5)$$

These equations cannot be solved exactly but numerical solutions by the application of the method of *Regula Falsi* (mentioned in paper I), may be obtained. Let us plot the angles θ_A and θ_B against D as shown in fig. 7(b). It is clear that since $\theta_B < \theta_A$ the $\theta_B(D)$ curve lies below the $\theta_A(D)$ curve. If the point (D, θ) lies below ths $\theta_B(D)$ curve then evidently $Z(D, \theta, \alpha)$ for all slit angles $\alpha_A > \alpha > \alpha_B$ will be $> \lambda_+$. Thus in order that condition (2.10b) be satisfied, it is essential that the electron must have its point (D, θ) in fig. 7(b) between $\theta=\pi/2$ line and the $\theta_B(D)$ curve. Conditions (2.10b) and (4.1) will be simultaneously satisfied if and only if an electron has its point (D,θ) within the area [marked off by thick lined boundaries in fig 7(b)] surrounded by $\theta=\pi/2$ line and the θ_i, θ_B and θ_s curves. The $\theta_B(D)$ curve cuts the θ_s curve at $D=D_{sB}$ where D_{sB} is given by

$$D_{sB}^2 = \frac{\lambda_+^2}{[\alpha_B + \sin^{-1}(s_B/2R_s)]^2} + (2R_s)^2. \quad (5.6)$$

It is immediately clear that if $D < 2R_i$ or $D > D_{sB}$ then no electron can pass through the positive portion of S_2.

Let us examine the curves in fig. 7(b) again in detail. For $\pi/2 > \theta > \theta_A$ we have $Z(\theta, \alpha)$ for all slit angles $\alpha_A > \alpha > \alpha_B$ will be $< \lambda_+$ and so if the azimuth of an electron (D, θ) lie within proper limits discussed in §3 and §4, all these electrons will be admitted by S_2. But difficulties arise for $\theta_A > \theta > \theta_B$. Let α_1 be the slit angle, corresponding to which $Z(\theta, D, \alpha_1)$ for any particular θ in $\theta_A > \theta > \theta_B$ is λ_+. Then $Z(\theta, D, \alpha) \gtrless \lambda_+$ according as $\alpha_A > \alpha > \alpha_1$ or $\alpha_1 > \alpha > \alpha_B$. Thus in order that condition (2,10b) be satisfied we must have $\alpha_1 > \alpha > \alpha_B$. α_1 is given by

$$2\alpha_1 = \frac{\lambda_+}{D\cos\theta} - \sin^{-1}\frac{2\delta}{D\sin\theta} - \sin\left(\frac{\lambda_+}{D\cos\theta}\right) \quad (5.7a)$$

Thus Φ_{α_1} = the azimuth of the electron (D, θ) for which the circle P passes through the point $M(\alpha=\alpha_1)$ in the line J, is given by

$$\Phi_{\alpha_1} = \frac{\lambda_+}{D\cos\theta}. \quad \ldots \quad (5.7b)$$

Thus if an electron (D, θ) has to pass through the counter slit, its azimuth ϕ should not lie in $\Phi_A > \phi > \Phi_{\alpha_1}$, but must lie in $\Phi_{\alpha_1} > \phi > \Phi_E$. This must be borne in mind in applying the results of §3 and §4 to electrons (D, θ) for θ in $\theta_A > \theta > \theta_B$.

The discussion for the negative position of the slit S_1, where $\pi/2 < \theta < \pi$, follows closely what has given above. The corresponding conclusions may be immediately drawn by an examination of the upper half of the fig. 7(*b*).

§6. TRANSMISSION FACTOR OF THE SPECTROMETER

We may now proceed to calculate the number of electrons entering the counter space in a magnetic field H. Let $n(p)$ be the distribution function in p, *i.e.*, $n(p)\,dp$ is the number of electrons emitted per second from one unit area of the sample plate per unit solid angle, having their momenta in the range p, $p+dp$. Then the number of electrons entering the counter space is evidently

$$N(H) = \int n(p)\,dp \int d\eta \int d\xi \int \sin\theta d\theta \int d\phi. \quad (6.1)$$

Let

$$\left.\begin{aligned} &T_1(p, \eta) = \int d\phi, \quad T_2(D, \xi, \eta) = \int T_1 \sin\theta d\theta, \\ &T_3(D, \eta) = \int T_2 d\xi, \quad T(D) = \int T_3 d\eta \end{aligned}\right\} \quad (6.2)$$

The limits of the integrals T_1, T_2 have already been discussed in §3, §4 and §5. We shall now numerically compute these integrals for the concrete case of the Lawson-Tyler β-ray spectrometer. The functions $T_1(p, \eta)$ have already been calculated in §4 for three different values of η, *viz.*, $\eta = -.15$, 0, $+.15$ cm. [*cf.* fig. 6(*b*)].

To calculate $T_2(D, \xi, \eta)$ we proceed as follows. Taking $\eta = 0$ cm., we first plot the θ_A, θ_B v.s. D curves for three different values of ξ, *viz.*, 0 cm., .4 cm., and .8 cm. Since θ is very nearly $\pi/2$ we may use the following approximation instead of using the method of *Regula Falsi*. Taking

$$\epsilon = \pi/2 - \theta, \quad \ldots \quad (6.3)$$

where ϵ is understood to be small, we get

$$\epsilon_A = \frac{\lambda}{D[\alpha_A + \sin^{-1}(s_A/D)]}, \quad \epsilon_B = \frac{\lambda}{D[\alpha_B + \sin^{-1}(s_B/D)]}. \quad \ldots \quad (6.4)$$

In fig. 7 (*c*). ϵ_A, ϵ_B have been plotted for the following values

$\lambda_+ = .8$ cm., .4 cm., 0 cm., $\lambda_- = .8$ cm., 1.2 cm., 1.6 cm.

It is quite evident that $\epsilon_B(D)$ are more or less constants. Values of ϵ for which the electron (D, ϵ) enters the counter is so small that $\sin\theta = \cos\epsilon \simeq 1$ and so ρ does not change appreciably as we change ϵ between the limits of the integral T_2. This means that the quantity T_1 in the integral T_2 may be taken constant and so taken out of the integration sign. Thus

$$T_2 = T_1[\epsilon_B(\lambda_+) + \epsilon_B(\lambda_-)]. \quad \dots \quad (6.5a)$$

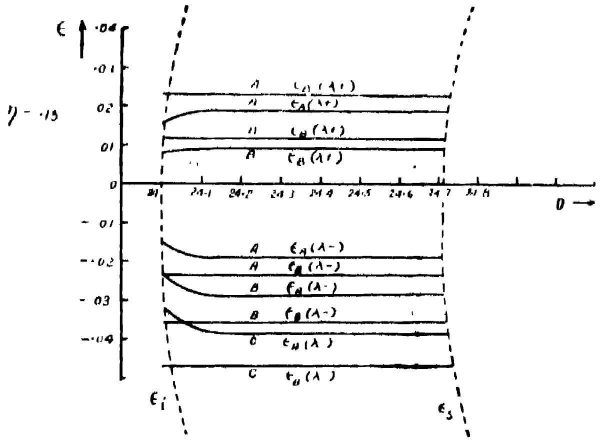
It is also evident from the [fig. 7(c)] that

$$\epsilon_B(\lambda_+) + \epsilon_B(\lambda_-) = .047 \text{ cm.} \quad \dots \quad (6.5b)$$

for all the three different values of ξ fixed upon. We have neglected the correction on the azimuth limits due to length of the counter slit parallel to the magnetic field, which, as mentioned at the end of §5, should be applied to those electrons D whose ϵ lie between ϵ_A and ϵ_B. This correction is, in fact, quite small. If we take the two other values of η, we find that the quantity (6.5a) above does not change perceptibly. Thus

$$T_3(D, \eta) = 1.6 \times .047 \times T_1(D, \eta). \quad \dots \quad (6.6)$$

It has already been shown in §4 that the function $T_1(D, \eta)$ for different values of η may be obtained by shifting $T_1(D, \eta = 0)$ curve by an amount η. Thus we



The curves A, B, C correspond respectively to $\xi = 0, .4, .8$ cm.

FIG. 7(c)

may employ the approximation (4.28) without introducing much error. As mentioned in §5, the proper superior limit to D for any particular point (ξ, η) on the sample plate is D_{sB}. But since the θ_B curve falls extremely steeply for θ near $\pi/2$ [$c.f.$ fig. 7(c)], D_{sB} is practically equal to $2R_s$. Thus it is clear that D must lie in $2R_i$ $(\eta = -.15) < D < 2R_s$ $(\eta = +.15)$ if the electron D has to enter the counter space. If $2R_i$ $(\eta = -.15) = 2R_1$, and $2R_s$ $(\eta = +.15) = 2R_2$, we have

$$2R_1 = D_1 = 23.7 \text{ cm.}, \quad 2R_2 = D_2 = 24.71 \text{ cm.} \quad \dots \quad (6.9)$$

For $2R_1 < D < 2R_i$ $(\eta = +.15)$ we have to calculate the area under the $T_1(D,$

$\eta=0$) curve between $2R_1(\eta=0)$ and $D+.15$ cm. in order to find $T(D)$, whereas for $2R_1(\eta=-.15) < D < 2R_2$ we have to find the area under the $T_1(D, \eta=0)$ curve

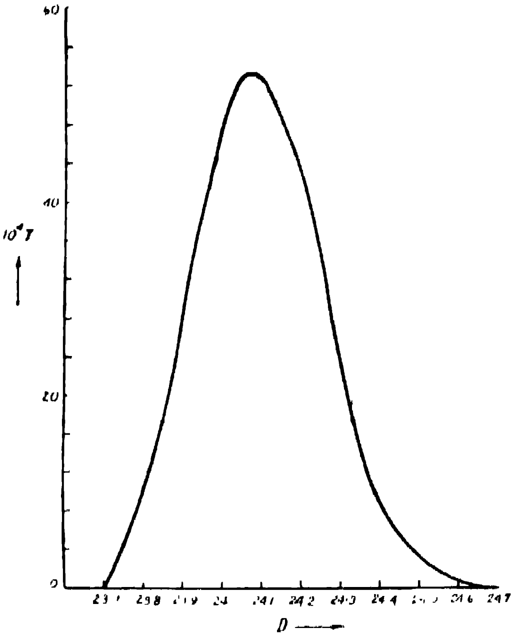


FIG. 8

between $D-.15$ cm and $2R_1(\eta=0)$. For D in $2R_1(\eta=+.15) < D < 2R_1$ $(\eta=-.15)$ we have however to calculate the area under the $T_1(D, \eta=0)$ curve between $D-.15$ cm and $D+.15$ cm. In this way, the $T(R)$ curve has been constructed [*cf.* fig. 8]. This function may be called the transmission factor function of the spectrometer. Our curve may be compared with the one deduced by Lawson and Tyler. We have finally

$$N(H)=\int_{p_1(H)}^{p_2(H)} n(p)T(p/H)dp. \qquad (6.10)$$

where $$p_1(H)=D_1H/2, \quad p_2(H) = D_2H/2. \qquad (6.11)$$

Equation (6.10) is an integral equation which must be solved in order that the momentum spectrum may be deduced from the experimental $N(H)$ curve.

§7. SOLUTION OF THE INTEGRAL EQUATION (6.10) OF THE SPECTROMETER

To complete the task of this paper we have now to show how the integral equation (6.10) may be solved. Lawson and Tyler have given an extremely elegent way of doing this Their method involves practically no labour. The best that we can do, therefore, is to transcribe their method in our notation. This is done in what follows. Since the range (p_1, p_2) is quite small we may develope $n(p)$ about some point in this range by Taylor's theorem. We have thus

$$n(p)=n(P)+\lambda(p-P)+\mu(p-P)^2+\text{ etc.} \quad \cdots \quad (7.1)$$

Substituting this in equation (6.10) we get

$$n(P)=\frac{N(H)}{HK}-\frac{\lambda H}{2K}\int(D-D_3)T(D)dD-\frac{\mu H^2}{4K}\int(D-D_3)^2T(D)dD-\text{etc.}$$

where

$$K=\tfrac{1}{2}\int_{D_1}^{D_2}T(D)dD, \quad \cdots \quad (7.2a)$$

and

$$D_3=\frac{2P}{H}. \quad \cdots \quad (7.2b)$$

Now let D_3 be chosen in such a way that

$$\int_{D_1}^{D_2}(D-D_3)T(D)dD=0. \quad \cdots \quad (7.3)$$

This equation may be solved graphically and the solution D_3 is evidently a constant of the spectrometer, like the quantities D_1 and D_2. For the transmission factor function, which we have calculated, we obtain $D_3=24.1036$ cm., whereas Lawson and Tyler obtain $D_3=24.1264$ cm. We have finally

$$n(P)=\frac{N(H)}{HK}-\frac{\mu}{K}\left(\frac{H}{2}\right)^2\int_{D_1}^{D_2}(D-D_3)^2T(D)dD-\text{etc.} \quad \cdots \quad (7.4)$$

If μ is small, as is in general the case, the correction term in above may be neglected, and

$$n(P)=\frac{N(H)}{HK}. \quad \cdots \quad (7.5)$$

This method fails when the maximum momentum of the electrons fall within the limits p_1 and p_2. Let H_1 be defined by

$$p_2(H_1)=\frac{D_2H_1}{2}=p_{max}. \quad \cdots \quad (7.6a)$$

Corresponding P is given by

$$P(H_1)=P_1=\frac{D_3H_1}{2}=\frac{D_3}{D_2}\,p_{max}. \quad \cdots \quad (7.6b)$$

Thus for fields $H<H_1$, the Lawson Tyler method will be applicable on account of the fact that $p_{max}>p_2(H)$. Consequently the momentum spectrum may be built up by the application of formula (7.5) up to $p\leq P_1$. On the other hand for $H>H_1$, since $p_2(H)>p_{max}$ the formula (7.5) is not applicable. According to Lawson-Tyler's calculations

$$D_3/2=12.0632\text{ cm}, \; D_2/2=12.3\text{ cm},$$

so that

$$\frac{D_2-D_3}{D_2}\simeq .018.$$

Thus the length of the tail of momentum spectrum ($P_1 < p < p_{max}$) over which the Lawson-Tyler method is inapplicable is about 1.8% of p_{max}. Over this tail the integral equation (6.10) takes the form

$$N(H) = \int_{p_1(H)}^{p_{max}} n(p)T(p/H)dp. \quad \dots \quad (7.7)$$

Let

$$D_{max} = \frac{2\, p_{max}}{H}. \quad \dots \quad (7.8)$$

Then the Lawson-Tyler's formula (7.4) may be made applicable over this tail also, if we give to the symbols P, D_3, K new meanings. Thus D_3 is to be determined as the solution of the equation

$$\int_{D_1}^{D_{max}} (D - D_3)T(D)dD = 0. \quad \dots \quad (7.9)$$

This equation may be solved graphically for different values of D_{max} and D_3 v.s. D_{max} curve may be plotted. P will be connected to D_3 by the old formula (7·2b). K will now have the meaning

$$K = \int_{D_1}^{D_{max}} T(D)dD. \quad \dots \quad (7.10)$$

To conclude, we make some remarks about the maximum momentum p_{max} This is determined from the upper limit of the $N(H)$ curve by

$$p_{max} = \frac{D_1 H_{max}}{2}. \quad \dots \quad (7.11)$$

It is however very difficult to determine H_{max} for generally, unless the source strength is very great, the $N(H)$ curve approaches zero very gradually. Thus the observed end point H'_{max} is in general different from the true end point H_{max}. In fact H'_{max} corresponds to that momentum p_1 for which the integral

$$\int_{p_1(H'_{max})}^{p_{max}} n(p)T(D)dp.$$

is indistinguishable from zero. Thus in order to determine p_{max} more accurately it is necessary to increase $n(p)$ by increasing the source strength, or to increase the transmission factor, If we gradually increase the source strength the observed end point H'_{max} will gradually increase until after some definite critical source strength, H'_{max} will remain stationary at some value, which we may take as the true end point. The end point may however be determined much more accurately and conveniently with a screen cathode β-ray spectrometer (Paper I) on account of the fact that the transmission factor of this spectrometer is of a much higher order of magnitude.

We have now to show how the positions and the intensities of the homogeneous groups of the conversion electrons may be determined. To do this we must remember that the transmission factor function $T(D)$ may also be interpreted as proportional to the $N(H)$ curve of a group of electrons of a single momentum. Thus if the momentum of any homogeneous group of electrons be p and the intensity of this group be I, *i.e.* if I such electrons be emitted per second per unit solid angle, per square cm. of the sample plate, then the $N(H)$ curve of these electrons will be given by

$$N(H) = IT(p/H).$$

Let H_1, H_2, H_3 be defined by

$$H_1 = \frac{2p}{D_1}, \quad H_2 = \frac{2p}{D_2}, \quad H_3 = \frac{2p}{D_4}$$

where D_1, D_2 have the same meaning as before and D_4 is the abscissa corresponding to which $T(D)$ has the maximum value. Then the $N(H)$ curve starts from zero at $H = H_1$ rises to a maximum at $H = H_3$ and then falls to zero at $H = H_2$. Thus if we can find accurately H_3 and $N(H_3)$, then it is possible to determine p and I. Evidently it is necessary to distinguish with great precision the $N(H)$ curve due to the β-rays with continuous spectrum from that due to the conversion electrons, and this can quite easily be done in practice.

An $M-F$ β-ray spectrometer with variable field is under construction in this laboratory. It is being planned by Mr. S. Das, M.Sc., in collaboration with the writer to study the β-radiations emitted from Co^{60} supplied to us by the courtesey of the M.I.T. cyclotron laboratory. The writer wishes to express his thanks to Prof. M. N. Saha, D.Sc., F.R.S. for his kind interest in the progress of this work. He thanks also Mr. S. Das, M.Sc. for his kind assistance in carrying out the numerical calculations of this paper.

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REFERENCES

Lawson, J. L. and Tyler, A. W. (1940), *Rev. Sci. Inst.*, **11**, 6.
Saha, A. K. (1944), *Proc. Nat. Inst. Sci, India*, **10**, 355.